3-D co-geometry 2

1. Given the equation of a straight line is given as $2(x+2) = 2(y-3) = z+1$.
   (a) Determine the vector equation of the straight line.
   (b) Hence, find the coordinates of a point Q that lies on the straight line such that $|OQ| = 3\sqrt{2}$.

2. Find the Cartesian equation of the plane passing through the points $A(4,-1,3)$ and $B(5,1,2)$ and containing the line : $\boldsymbol{r} = 4\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k} + \lambda(3\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k})$.

3. The position vectors of the points P, Q and R are $\boldsymbol{i} + 3\boldsymbol{k}, 2\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}, \boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ respectively.
   Let the plane π contains the points P, Q and R.
   (a) Find a vector which is perpendicular to plane π.
   (b) Find the area of ΔPQR.
   (c) Obtain the Cartesian equation of plane π.

4. Find the vector $\boldsymbol{n}_1$ normal to the plane $\pi_1 : \boldsymbol{r} = (5\boldsymbol{i} + \boldsymbol{j}) + \alpha(-4\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}) + \beta(\boldsymbol{j} + 2\boldsymbol{k})$.
   Write down a vector $\boldsymbol{n}_2$ normal to the plane $\pi_2 : 3x + y - z = 3$. Show that $4\boldsymbol{i} + 13\boldsymbol{j} + 25\boldsymbol{k}$ is normal to both $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$. Given that the point $(1,1,1)$ lies on both π_1 and π_2.
   Write down the equation of line of intersection of π_1 and π_2 in the form of $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ where t is a parameter.

5. $\boldsymbol{OA} = \boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k},\ \boldsymbol{OB} = 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}, \boldsymbol{OC} = 3\boldsymbol{i} + \boldsymbol{j}, \boldsymbol{OD} = 2\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k}$.
   Point P divides the line AC in the ratio 2 : 1 internally.
   (a) Show that ABCD is a parallelogram.
   (b) Calculate the exact area of the parallelogram ABCD.
   (c) Find the position vector P and the angle APB in degrees corrected to one decimal place.

6. Show that the lines with equations
   $\boldsymbol{r} = 7\boldsymbol{i} - 3\boldsymbol{j} + 3\boldsymbol{k} + \lambda(3\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k})$ and $\boldsymbol{r} = 7\boldsymbol{i} - 2\boldsymbol{j} + 4\boldsymbol{k} + \mu(-2\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k})$ intersect,
   and find the position vector of their point of intersection.

7. Given a sphere $x^2 + y^2 + z^2 = 126$
   (i) Find the equations of the tangent planes to the sphere when $x = 1$ and $y = 10$.
   (ii) Find a point on the sphere that is farthest to the point (1, 2, 3). Hence, determine its distance.